

# Review of Fractions

**1. Adding Fractions with a Common Denominator:** Add the numerators.

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

but avoid this common error

$$\frac{a}{b+c} + \frac{a}{b} = \frac{a}{c}$$

Examples:  $\frac{1}{7} + \frac{3}{7} = \frac{1+3}{7} = \frac{4}{7}$

$$\frac{1}{x} + \frac{3}{x} = \frac{1+3}{x} = \frac{4}{x}$$

**2. Adding Fractions with Different Denominators:** Obtain a common denominator and add numerators.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

Examples:  $\frac{2}{3} + \frac{1}{5} = \frac{2 \cdot 5}{3 \cdot 5} + \frac{3 \cdot 1}{3 \cdot 5} = \frac{10+3}{15} = \frac{13}{15}$

$$\frac{2}{x} + \frac{1}{y} = \frac{2 \cdot y}{x \cdot y} + \frac{x \cdot 1}{x \cdot y} = \frac{2y+x}{xy}$$

**3. Multiplying Fractions:** Multiply their numerators and multiply their denominators.

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Examples:  $\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$

$$\frac{2a}{x} \cdot \frac{4a}{y} = \frac{2a \cdot 4a}{x \cdot y} = \frac{8a^2}{xy}$$

**4. Reducing Fractions:** Factor numerator and denominator and cancel common factors.

$$\frac{ac}{bc} = \frac{a\cancel{c}}{b\cancel{c}} = \frac{a}{b}$$

Examples:  $\frac{6}{15} = \frac{2 \cdot 3}{5 \cdot 3} = \frac{2\cancel{3}}{5\cancel{3}} = \frac{2}{5}$

$$\frac{2}{4} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{1\cancel{2}}{2\cancel{2}} = \frac{1}{2}$$

$$\frac{8xyz}{12axz} = \frac{2 \cdot 4 \cdot x \cdot y \cdot z}{3 \cdot 4 \cdot a \cdot x \cdot z} = \frac{2\cancel{4} \cdot \cancel{x} \cdot y \cdot \cancel{z}}{3 \cdot \cancel{4} \cdot a \cdot \cancel{x} \cdot \cancel{z}} = \frac{2y}{3a}$$

**5. Negative Fraction:** The negative sign can be applied to the numerator or to the denominator or to the fraction as a whole.

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Examples:  $\frac{-2}{3} = \frac{2}{-3} = -\frac{2}{3}$

$$\frac{a}{-a} = -\frac{a}{a} = -\frac{\cancel{a}}{\cancel{a}} = -1$$

$$\frac{x-3}{3-x} = \frac{x-3}{-(x-3)} = -\frac{x-3}{x-3} = -\frac{\cancel{x-3}}{\cancel{x-3}} = -1$$

## Review of Fractions

**6. Dividing Fractions:** Invert the divisor (second fraction) and multiply.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Example:  $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{2 \cdot 7}{3 \cdot 5} = \frac{14}{15}$

**7. Dividing Fractions:** Invert the divisor (bottom fraction) and multiply.

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Examples:  $\frac{\frac{2}{3}}{\frac{5}{7}} = \frac{2}{3} \cdot \frac{7}{5} = \frac{2 \cdot 7}{3 \cdot 5} = \frac{14}{15}$

$$\frac{\frac{2x}{3z}}{\frac{5x}{7y}} = \frac{2x}{3z} \cdot \frac{7y}{5x} = \frac{2 \cdot 7 \cdot \cancel{x} \cdot y}{3 \cdot 5 \cdot \cancel{x} \cdot z} = \frac{14y}{15z}$$

**8. Any Number is a Fraction:** Any number (or expression) can be made a fraction with a denominator of 1.

$$a = \frac{a}{1}$$

Examples:  $4 = \frac{4}{1}$

$$\pi = \frac{\pi}{1}$$

$$x = \frac{x}{1}$$

$$x - 3 = \frac{x - 3}{1}$$

**9. Simplifying Compound Fractions:** Using the rules above, first combine the terms of the numerator into a single fraction. Second, combine the terms of the denominator into a single fraction. Third, invert and multiply.

Example:  $\frac{\frac{x}{y} + 2}{\frac{1}{x} - y}$

1)  $= \frac{\frac{x}{y} + \frac{2}{1}}{\frac{1}{x} - y} = \frac{\frac{x}{y} + \frac{2y}{y}}{\frac{1}{x} - y} = \frac{\frac{x + 2y}{y}}{\frac{1}{x} - y} = \frac{\frac{x + 2y}{y}}{\frac{1}{x} - y}$

2)  $= \frac{\frac{x + 2y}{y}}{\frac{1}{x} - y} = \frac{\frac{x + 2y}{y}}{\frac{1}{x} - \frac{xy}{x}} = \frac{\frac{x + 2y}{y}}{\frac{1 - xy}{x}}$

3)  $= \frac{\frac{x + 2y}{y}}{\frac{1 - xy}{x}} = \frac{x + 2y}{y} \cdot \frac{x}{1 - xy} = \frac{x(x + 2y)}{y(1 - xy)}$